

Target Space Dualities of Heterotic Grand Unified Theories

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ABSTRACT. In this article we summarize and extend the ideas and investigations on so called target space dualities of heterotic models with $(0, 2)$ world-sheet supersymmetry as they were partly presented on the String-Math 2011 conference. After the generic description of the duality, we give some novel examples involving vector bundles that are not deformations of the tangent bundle but more generic ones corresponding to $SO(10)$ and $SU(5)$ gauge theories in four dimensions. We show explicitly that the necessary conditions for a duality also hold for compactifications of this kind. Finally we will present the results of the large landscape scan of E_6 models.

1. Introduction

Heterotic string theory provides a way to build grand unified theories very naturally. A priori it arises with an $E_8 \times E_8$ gauge group which can then be broken down to some GUT group in four dimensions. In such a model, on the one hand, we have bosonic degrees of freedom that are valued in the Calabi-Yau manifold, denoted by \mathcal{M} throughout this paper, and on the other hand their fermionic superpartners that are coupled to the pullback of the tangent bundle. Furthermore we have some left-moving fermions that do not necessarily couple to the pullback of the tangent bundle but more generically to the pullback of some vector bundle \mathcal{V} of rank $n = 3, 4$ or 5 . The only conditions on this vector bundle is that it is holomorphic, stable and that the first and second Chern classes equal those of the tangent bundle. The structure group of \mathcal{V} is then $SU(3)$, $SU(4)$ or $SU(5)$ for a rank 3 , 4 or 5 bundle respectively and breaks one of the E_8 factors down to the commutant of $SU(n)$ in E_8 which gives an effective four-dimensional theory with gauge group E_6 , $SO(10)$ or $SU(5)$ respectively whereas the other E_8 factor can be hidden. In the geometric picture the massless matter spectrum can be obtained by computing certain vector bundle valued cohomology classes. In case of an E_6 , $SO(10)$ or $SU(5)$ gauge group the actual cohomologies that need to be computed can be found in table 1. A review on the computational tools for such a kind of calculation can be found e.g. in [1] and references therein.

# zero modes in reps of $H \times G$	1	$h_{\mathcal{M}}^1(\mathcal{V})$	$h_{\mathcal{M}}^1(\mathcal{V}^*)$	$h_{\mathcal{M}}^1(\Lambda^2 \mathcal{V})$	$h_{\mathcal{M}}^1(\Lambda^2 \mathcal{V}^*)$	$h_{\mathcal{M}}^1(\mathcal{V} \otimes \mathcal{V}^*)$
E_8				248		
\downarrow				\downarrow		
$SU(3) \times E_6$	$(1, 78) \oplus (3, 27)$	$\oplus (\bar{3}, \bar{27})$				$\oplus (8, 1)$
$SU(4) \times SO(10)$	$(1, 45) \oplus (4, 16)$	$\oplus (\bar{4}, \bar{16})$	$\oplus (6, 10)$			$\oplus (15, 1)$
$SU(5) \times SU(5)$	$(1, 24) \oplus (5, \bar{10})$	$\oplus (\bar{5}, 10)$	$\oplus (10, 5)$	$\oplus (\bar{10}, \bar{5})$		$\oplus (24, 1)$

TABLE 1. Matter zero modes in representations of the GUT group

One nice way to see how holomorphic vector bundles arise in string-theory is using the $(0, 2)$ gauged linear sigma model (GLSM) [2]. Here one starts with a two dimensional gauge theory containing certain chiral as well as Fermi superfields. The most general superpotential of such a theory can be written as

$$(1) \quad S = \int d^2 z d\theta \left[\sum_j \Gamma^j G_j(X_i) + \sum_{l,a} P_l \Lambda^a F_a{}^l(X_i) \right],$$

where the X_i , P_l and Γ^j , Λ^a correspond to chiral and Fermi superfields. They are all charged under a certain number of $U(1)$ gauge groups in such a way that the superpotential is gauge invariant. If we denote the bosonic components of the chiral superfields X_i and P_l by x_i and p_l we will find a bosonic potential for them consisting of an F-term potential

$$(2) \quad V_F = \sum_j \left| G_j(x_i) \right|^2 + \sum_a \left| \sum_l p_l F_a{}^l(x_i) \right|^2$$

as well as a D-term scalar potential

$$(3) \quad V_D = \sum_{\alpha=1}^r \left(\sum_{i=1}^d Q_i^{(\alpha)} |x_i|^2 - \sum_{l=1}^{\gamma} M_l^{(\alpha)} |p_l|^2 - \xi^{(\alpha)} \right)^2,$$

where the $\xi^{(\alpha)} \in \mathbb{R}$ denote the Fayet-Iliopoulos (FI) parameters for each $U(1)$ gauge group. One can now go ahead and analyze the vacuum structure of such a theory and certainly this structure will crucially depend on the actual value of the FI parameters. Putting more weight on the mathematical point of view, one calls them also Kähler parameters. Different choices of these parameters can be interpreted as different triangulations of some polytope and each triangulation describes one kind of vacuum or phase of the underlying GLSM [3]. If one chooses them in such a way that the triangulation is maximal, i.e. it has more or the same number of maximal dimensional cones than every other possible triangulation then our vacuum may correspond to a holomorphic vector bundle over a Calabi-Yau manifold. In the low energy effective action this will be the corresponding target space of the non-linear sigma model.

Here the idea of target space dualities comes into play. It basically works with the phases of the GLSM that do not have a completely geometric interpretation and uses some freedom in them to change the GLSM without spoiling the phase itself. This basically means that the moduli spaces of two GLSMs can be connected and that there exists a locus inside a specific phase where they coincide. After the

redefinition of the data, we can just pretend that we came from the new GLSM and go back to the geometric phase there. The question is now whether we actually undergo a transition of one geometry to a different one by doing this or if we just walk around in one and the same moduli space and are dealing with a target space duality between those two geometries. Distler and Kachru first pointed out that such a thing might actually exist [4] and further work [5–7] supported this idea with specific examples by explicitly comparing the dimensions of the moduli spaces as well as the spectrum of the corresponding models and found agreement:

$$\begin{aligned} h^{1,1}(\mathcal{M}) + h^{2,1}(\mathcal{M}) + h_{\mathcal{M}}^1(\text{End}(\mathcal{V})) &= h^{1,1}(\widetilde{\mathcal{M}}) + h^{2,1}(\widetilde{\mathcal{M}}) + h_{\widetilde{\mathcal{M}}}^1(\text{End}(\widetilde{\mathcal{V}})) , \\ h_{\mathcal{M}}^i(\wedge^k \mathcal{V}) &= h_{\widetilde{\mathcal{M}}}^i(\wedge^k \widetilde{\mathcal{V}}), \quad \text{for } i = 0, \dots, 3. \end{aligned}$$

2. The Duality

In this section we will review the general procedure that can be employed to produce dual models from a given one. In order to do that we first explain how this can be done schematically and then turn to three explicit and novel examples. Here we will capture all possible structure groups for the bundle, i.e. we consider an $SU(3)$, $SU(4)$ and at last an $SU(5)$ bundle. That the duality works for such models was mentioned in [8] but not shown in an explicit example. We want to use the opportunity to catch up on that.

2.1. General procedure. In the setting of the GLSM we mentioned above we can specify the vector bundle as the cohomology of a complex containing direct sums of line bundles

$$(4) \quad 0 \rightarrow \mathcal{O}_{\mathcal{M}}^{\oplus r_{\mathcal{V}}} \xrightarrow{\otimes E_i^a} \bigoplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a^l} \bigoplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \rightarrow 0.$$

This means that the vector bundle is given by

$$(5) \quad \mathcal{V} = \frac{\ker(F_a^l)}{\text{im}(E_i^a)}.$$

The chiral and Fermi fields of the corresponding GLSM are charged under the various C^* actions of the toric variety that contains \mathcal{M} as complete intersection of hypersurfaces. In the following we will denote the defining data of the configuration by two tables

x_i	Γ^j	Λ^a	p_l
$Q_1 \quad \dots \quad Q_N$	$-S_1 \quad \dots \quad -S_c$	$N_1 \quad \dots \quad N_{\delta}$	$-M_1 \quad \dots \quad -M_{\gamma}$

In this scenario we can turn the requirement of matching first and second Chern classes of \mathcal{V} and $T_{\mathcal{M}}$ into a combinatorial relation of the charges of the fields:

$$(6) \quad \begin{aligned} \sum_{a=1}^{\delta} N_a^{(\alpha)} &= \sum_{l=1}^{\gamma} M_l^{(\alpha)}, & \sum_{i=1}^d Q_i^{(\alpha)} &= \sum_{j=1}^c S_j^{(\alpha)}, \\ \sum_{l=1}^{\gamma} M_l^{(\alpha)} M_l^{(\beta)} - \sum_{a=1}^{\delta} N_a^{(\alpha)} N_a^{(\beta)} &= \sum_{j=1}^c S_j^{(\alpha)} S_j^{(\beta)} - \sum_{i=1}^d Q_i^{(\alpha)} Q_i^{(\beta)}. \end{aligned}$$

Here the Greek index in parenthesis labels the corresponding C^* action of the toric variety and the equations (6) have to be satisfied for all α and β . Now after fixing

the notations we can go on to describe the explicit procedure which consists of different steps:

The procedure:

- (1) Construct the GLSM phases of a smooth $(0, 2)$ model $(\mathcal{M}, \mathcal{V})$.
- (2) Go to a phase where one of the p_l , say p_1 , is not allowed to vanish and hence obtains a vev $\langle p_1 \rangle$.
- (3) Perform a rescaling of k Fermi superfields by the constant vev $\langle p_1 \rangle$ and exchange the role of some Λ^a and Γ^j

$$\tilde{\Lambda}^{a_i} := \frac{\Gamma^{j_i}}{\langle p_1 \rangle}, \quad \tilde{\Gamma}^{j_i} := \langle p_1 \rangle \Lambda^{a_i}, \quad \forall i = 1, \dots, k,$$

with $\sum_i \|G_{j_i}\| = \sum_i \|F_{a_i}^{-1}\|$ for anomaly cancellation.

- (4) Move to a region in the bundle moduli space where the Λ^{a_i} only appear in terms with P_1 for all i . This means that we choose the coefficients in the bundle defining polynomials F_a^l such that

$$F_{a_i}^l = 0, \quad \forall l \neq 1, \quad i = 1, \dots, k.$$

- (5) Leave the non-geometric phase and define the Fermi superfields of the new GLSM such that each term in the superpotential is $U(1)^r$ gauge invariant. This means

$$\|\tilde{\Lambda}^{a_i}\| = \|\Gamma^{j_i}\| - \|P_1\| \quad \text{and} \quad \|\tilde{\Gamma}^{j_i}\| = \|\Lambda^{a_i}\| + \|P_1\|.$$

- (6) Returning to a generic point in moduli space defines a new dual $(0, 2)$ GLSM which in a geometric phase corresponds to a different Calabi-Yau/vector bundle configuration $(\tilde{\mathcal{M}}, \tilde{\mathcal{V}})$.

2.2. Examples of models with structure group $SU(n)$. We now want to be a bit more explicit and show some specific examples of the duality. We will show one example for each type of structure group $SU(3)$, $SU(4)$ and $SU(5)$. In contrast to earlier work [8] where we mostly focused on models that were deformations of the tangent bundle and hence given by the cohomology of the Euler sequence, here we want to give different examples that do arise from an exact monad

$$(7) \quad 0 \rightarrow \mathcal{V} \xrightarrow{f} \bigoplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a^l} \bigoplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \rightarrow 0,$$

and hence are given by the kernel of the map F_a^l

$$(8) \quad \mathcal{V} = \text{Ker} (F_a^l).$$

The way to generate the dual models of such a monad remains the same.

Example for an $SU(3)$ -model: We start with an $SU(3)$ example which consists of a holomorphic vector bundle over a codimension two complete intersection Calabi-Yau space. In [8] we investigated mostly $SU(3)$ models which are given by a deformation of the tangent bundle. As described there the base space undergoes usually a conifold transition. In this example here we are not dealing with a $(0, 2)$ model which is a deformation of the tangent bundle but a completely independent monad. Furthermore we will see that the base will not transform via a conifold transition. Rather in the beginning the ambient variety will remain untouched and

only a different set of hypersurfaces will be chosen, also resulting in a topology change of the base. Finally through the exchange of those specific hypersurfaces we will see that in fact the ambient space topology will be changed after all. The model data is given by

$$(9) \quad \begin{array}{c|c} x_i & \Gamma^j \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 & -2 & -2 \\ 1 & 1 & 1 & 2 & 2 & 2 & 0 & -4 & -5 \end{array} \quad \begin{array}{c|c} \Lambda^a & p_l \\ \hline 1 & 0 & 0 & 2 & -3 \\ 0 & 1 & 1 & 6 & -8 \end{array}.$$

As was explained in [9] to compute the number of chiral matter zero modes and the massless singlets, we need nothing but line bundle cohomologies. Furthermore the efficient algorithm suggested in [10] and proven in [11] and [12] allows one to calculate such cohomologies quite fast. Employing our implementation **cohomCalc Koszul** extension [13] for this matter we find

$$(10) \quad h_{\mathcal{M}}^{\bullet}(\mathcal{V}) = (0, 120, 0, 0), \\ h_{\mathcal{M}}^{1,1} + h_{\mathcal{M}}^{2,1} + h_{\mathcal{M}}^1(\text{End}(\mathcal{V})) = 2 + 68 + 322 = 392.$$

In order to see our freedom of consistently exchanging hypersurface equations with bundle maps in the monad as described in the last section we explicitly write down the multi-degrees of the corresponding generic homogeneous functions. Using that

$$(11) \quad \|F_a^l\| = -\|p_l\| - \|\Lambda^a\|,$$

for the only choice $l = 1$ they read

$$(12) \quad \|G_1\| = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \|G_2\| = \begin{pmatrix} 2 \\ 5 \end{pmatrix},$$

$$(13) \quad \|F_1^1\| = \begin{pmatrix} 2 \\ 8 \end{pmatrix}, \|F_2^1\| = \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \|F_3^1\| = \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \|F_4^1\| = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Here we can already see that the sum of the degrees of the two hypersurfaces equals the sum of the degree of the third and the fourth F . From the last section, we know how to exchange these functions and how to redefine the Λ 's and Γ 's in order to obtain a sensible new monad. Namely we perform the rescalings

$$(14) \quad \begin{aligned} \tilde{\Gamma}^1 &:= \langle p_1 \rangle \Lambda^3, & \tilde{\Gamma}^B &:= \langle p_1 \rangle \Lambda^4, & \tilde{\Lambda}^3 &:= \frac{\Gamma^1}{\langle p_1 \rangle}, & \tilde{\Lambda}^4 &:= \frac{\Gamma^B}{\langle p_1 \rangle}, \\ \tilde{G}_1 &:= F_3^1, & \tilde{G}_2 &:= F_4^1, & \tilde{F}_3^1 &:= G_1, & \tilde{F}_4^1 &:= G_2, \end{aligned}$$

yielding the effective superpotential

$$(15) \quad \mathcal{W} = \tilde{\Gamma}^1 \tilde{G}_1 + \tilde{\Gamma}^2 \tilde{G}_2 + \langle p_1 \rangle \left(\tilde{\Lambda}^3 \tilde{F}_3^1 + \tilde{\Lambda}^4 \tilde{F}_4^1 + \Lambda^1 F_1^1 + \Lambda^2 F_2^1 \right).$$

The new charges of the constructed model read

$$(16) \quad \begin{aligned} \|\tilde{\Gamma}^1\| &= \begin{pmatrix} -3 \\ -7 \end{pmatrix}, & \|\tilde{\Gamma}^2\| &= \begin{pmatrix} -1 \\ -2 \end{pmatrix}, & \|\tilde{\Lambda}^3\| &= \begin{pmatrix} 1 \\ 4 \end{pmatrix}, & \|\tilde{\Lambda}^4\| &= \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \\ \|\tilde{G}_1\| &= \begin{pmatrix} 3 \\ 7 \end{pmatrix}, & \|\tilde{G}_2\| &= \begin{pmatrix} 1 \\ 2 \end{pmatrix}, & \|\tilde{F}_3^1\| &= \begin{pmatrix} 2 \\ 4 \end{pmatrix}, & \|\tilde{F}_4^1\| &= \begin{pmatrix} 2 \\ 5 \end{pmatrix} \end{aligned}$$

and hence going back to the geometric phase we obtain the new base with a new vector bundle. We notice that the new configuration can be rewritten in a slightly simpler way. The new hypersurface \tilde{G}_2 has precisely the same degree as the divisor $\{x_4 = 0\}$ and therefore the corresponding constraining equation to the ambient space simply removes this coordinate from the configuration and we obtain

$$(17) \quad \begin{array}{c|c} x_i & \Gamma^j \\ \hline 0 & 0 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 2 & 2 & 0 \\ \hline \end{array} \quad \begin{array}{c|c} \Lambda^a & p_l \\ \hline 1 & 0 & 1 & 1 \\ \hline 0 & 1 & 4 & 3 \\ \hline \end{array} \quad \begin{array}{c} -3 \\ -7 \end{array} \quad \begin{array}{c} -3 \\ -8 \end{array}.$$

As was generically shown, this configuration still satisfies the conditions (6) and we obtain the following topological data:

$$(18) \quad \begin{aligned} h_{\mathcal{M}}^{\bullet}(\tilde{\mathcal{V}}) &= (0, 120, 0, 0), \\ h_{\mathcal{M}}^{1,1} + h_{\mathcal{M}}^{2,1} + h_{\mathcal{M}}^1(\text{End}(\tilde{\mathcal{V}})) &= 2 + 95 + 295 = 392. \end{aligned}$$

If we compare this with the result we obtained in (10), we see that the number of chiral zero modes did not change and the total number of first order deformations stayed the same even though the Hodge number $h^{2,1}$ changed drastically.

Let us once more put some emphasis on the fact that we started up with a base manifold that was of codimension two and due to the exchange ended up with a simpler space given by a codimension one Calabi-Yau manifold. Similarly, as we will see in the next example this can also happen the other way round resulting in an increase of the codimension. Also the number of C^* actions can change which will be shown in the following examples, too.

An example for an $SU(4)$ -model: Next we present an example of a dual pair of heterotic $(0, 2)$ models that give rise to gauge group $SO(10)$ in four dimensions and hence are equipped with a rank 4 vector bundle. The model is again not a deformation of the tangent bundle. The base is the complete intersection of a generic quartic and homogeneous degree hypersurface two inside \mathbb{P}^5 . The defining data can be read off in the following table:

$$(19) \quad \begin{array}{c|c} x_i & \Gamma^j \\ \hline \mathbb{P}^5 & -2 & -4 \\ \hline \end{array} \quad \begin{array}{c|c} \Lambda^a & p_l \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline \end{array} \quad \begin{array}{c} -3 \\ -2 \\ -2 \end{array}.$$

Clearly this model is anomaly free, i.e. it satisfies (6) and one can also show that the vector bundle is also stable. It is sometimes also referred to as a positive monad, since all line bundles involved have positive degree. It has the following topological data:

$$(20) \quad \begin{aligned} h_{\mathcal{M}}^{\bullet}(\tilde{\mathcal{V}}) &= (0, 48, 0, 0), \\ h_{\mathcal{M}}^{1,1} + h_{\mathcal{M}}^{2,1} + h_{\mathcal{M}}^1(\text{End}(\tilde{\mathcal{V}})) &= 1 + 89 + 159 = 249. \end{aligned}$$

Before we move on, we introduce a new coordinate along with a new hypersurface to the model. Doing that at the same time does not change the model at all. In order to perform the exchange of polynomials F and G , we have to go to a certain region of the moduli space, exchange them and go back to the generic region in the dual configuration. The resulting base manifold can then be obtained as the

conifold transition of the initial base space. The full model is then given by

$$(21) \quad \begin{array}{c|c|c|c} x_i & \Gamma^j & \Lambda^a & p_l \\ \hline \mathbb{P}^1 & -1 & 0 & -1 \\ \mathbb{P}^5 & -1 & -4 & -1 \end{array} \quad \begin{array}{c|c|c|c} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array} \quad \begin{array}{c|c|c} 0 & -1 & 0 \\ -3 & -2 & -2 \end{array},$$

and its topology satisfies the necessary duality check of coinciding spectrum and moduli space dimensions:

$$(22) \quad h_{\mathcal{M}}^{\bullet}(\tilde{\mathcal{V}}) = (0, 48, 0, 0), \\ h_{\mathcal{M}}^{1,1} + h_{\mathcal{M}}^{2,1} + h_{\mathcal{M}}^1(\text{End}(\tilde{\mathcal{V}})) = 2 + 86 + 161 = 249.$$

An example for an $SU(5)$ -model: Finally let us quickly state a different bundle over the same base from the last paragraph. We modify it such that it has no longer $SU(4)$ but rather $SU(5)$ structure. It is given by

$$(23) \quad \begin{array}{c|c|c|c} x_i & \Gamma^j & \Lambda^a & p_l \\ \hline \mathbb{P}^5 & -2 & -4 & \end{array} \quad \begin{array}{c|c|c|c|c|c|c|c} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array} \quad \begin{array}{c|c|c} -3 & -3 & -2 \end{array}.$$

Since we have still three chiral fields p_l and eight Fermi fields Λ^a , we end up with a rank 5 vector bundle and hence with an $SU(5)$ gauge group in the four-dimensional theory. The spectrum and the dimension of the moduli space for this model can be calculated as

$$(24) \quad h_{\mathcal{M}}^{\bullet}(\tilde{\mathcal{V}}) = (0, 72, 0, 0), \\ h_{\mathcal{M}}^{1,1} + h_{\mathcal{M}}^{2,1} + h_{\mathcal{M}}^1(\text{End}(\tilde{\mathcal{V}})) = 1 + 89 + 288 = 378.$$

The dual base is again given by the same conifold transition as in the last paragraph. Altogether we get

$$(25) \quad \begin{array}{c|c|c|c} x_i & \Gamma^j & \Lambda^a & p_l \\ \hline \mathbb{P}^1 & -1 & 0 & -1 \\ \mathbb{P}^5 & -1 & -4 & -1 \end{array} \quad \begin{array}{c|c|c|c|c|c|c|c} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 \end{array} \quad \begin{array}{c|c|c} 0 & 0 & -1 \\ -3 & -3 & -2 \end{array}.$$

Calculating the topological data,

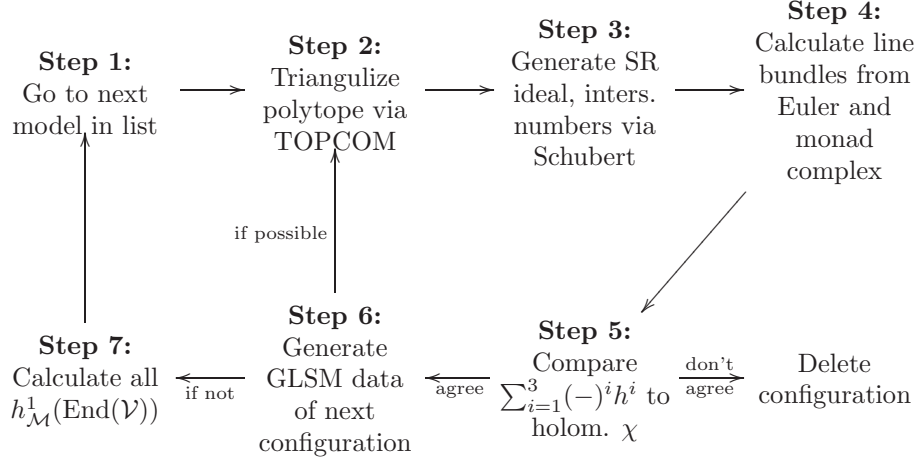
$$(26) \quad h_{\mathcal{M}}^{\bullet}(\tilde{\mathcal{V}}) = (0, 72, 0, 0), \\ h_{\mathcal{M}}^{1,1} + h_{\mathcal{M}}^{2,1} + h_{\mathcal{M}}^1(\text{End}(\tilde{\mathcal{V}})) = 2 + 58 + 318 = 378$$

we can verify that the necessary condition for a duality also holds and hence the conjecture extends to $SU(5)$ bundles as well.

3. Landscape Scan

In this section we will review the results that have been obtained in the large landscape scan in [8]. In contrast to the examples we just discussed, there we focused on models that arise as deformations of the tangent bundle of some Calabi-Yau subvariety in a toric geometry. Here we get the condition (6) as well as the bundle stability automatically which saves a lot of work.

The scanning algorithm: The algorithm that we used to scan through the large sets of configurations was programmed such that it runs through a given set of smooth configurations and produces combinatorially all dual configurations that one obtains by performing the procedure described in 2.1. We made sure to start with configurations that are smooth and have a stable bundle. But by constructing the dual models it may happen that new singularities are produced or also that the bundle is destabilized. We did not sufficiently check for these issues but rather made some necessary checks to test it. The algorithm can be summarized by the following chart:



We ran through two different lists (mentioned in step 1). The first one contained Calabi-Yau manifolds defined via single hypersurfaces in toric varieties. We took the ambient spaces out of the list from [14] available on the website of Maximilian Kreuzer [15] and the second list contains codimension 2 complete intersections in weighed projective spaces which is part of the list presented in [16] and available at [17]. To resolve the ambient spaces and also to generate the set of nef partitions to obtain the codimension 2 Calabi-Yaus, we used PALP [18]. For the remaining steps several packages as TOPCOM [19] Schubert [20] and of course **cohomCalc Koszul** extension [13] along with some Mathematica routines were employed. For the interplay of TOPCOM and Schubert we use the (not published) Toric Triangulizer [21].

The results: Our scan ran over the list of hypersurfaces in toric varieties [15] where we considered all toric varieties with 7, 8 and 9 lattice points which make altogether 1,085. Additionally we scanned over a large set of complete intersections of hypersurfaces in weighted projective spaces. This list can be found online at [17]. For our scan we simply ran through the first 2,780 ambient spaces and chose the 16,029 possible nef partitions as starting points. All these nef partitions correspond to topologically distinct Calabi-Yau manifolds that are complete intersections of two hypersurfaces in the corresponding weighted projective space. Starting from a codimension one Calabi-Yau we performed all first duals to each of those models in the way described in 2.1. Most of the dual models of each hypersurface Calabi-Yau were given by codimension 2 complete intersections in toric varieties. Since already many of the duals are obtained by only performing the duality procedure

Co-dim	Different classes	Possibly smooth models	Classes without duals	Models with matching spectrum	Models with full agreement
1	1,085	4,507	42	4,144 (100%)	1509 (94.6%)
2	16,961	79,204	718	64,332 (85 %)	20,336 (91%)

TABLE 2. Some data on the landscape study: The codimension is the one of the model we started with. The percent numbers in the parentheses only cover models where these numbers could actually be calculated. In column 6, by “full agreement” we mean that the chiral spectrum of dual models as well as the sum of their complex structure, Kähler and bundle deformations agree with the initial ones.

once, we did not perform duals of duals. Similarly for most of the dual geometries corresponding to a intersection of two hypersurfaces were complete intersections of three hypersurfaces in some toric variety. Some details on the the results can be found in table 2.

4. Conclusions

In this letter we reviewed and extended a method to construct from a given heterotic $(0, 2)$ model, dual models that generically have the same massless spectra. Continuing earlier work we explicitly showed that this procedure actually also works for models that are not deformations of the tangent bundle and may have $SU(3)$, $SU(4)$ or $SU(5)$ structure group by testing some necessary conditions for such models to be dual.

We also presented the results of the landscape scan from earlier work which includes many configurations that actually are all deformations of the tangent bundle. They arise as codimension one and codimension two complete intersections in toric varieties and weighted projective spaces respectively. This scan provided evidence for the fact that the proposed procedure generates dual configurations indeed. Having only tested single examples for the scenario where the model is not a deformation of the tangent bundle and therefore comes generically with $SU(n)$ structure for $n = 3, 4, 5$ it remains to perform a similar larger scan for such models as well. That we did not do it so far has a reasons. Namely it is first of all not very easy to solve (6) in general for a given base geometry. The second thing is that once one has a configuration, one has to check that the bundle is not singular and that it is furthermore stable. Since this is quite a challenge, we did not manage to systematically construct such models, yet. On the other hand, if one could come up with an idea to generate all stable bundles over a given base geometry systematically, it would be no problem to check (6) for those models. This was actually already done for a subset of all bundles over specific base spaces [22] and one way to prove bundle stability in an up to some point systematic way for arbitrary base spaces was suggested in [23] and [24] and gives hope to enable us to overcome this challenge.

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